$$\begin{cases} u_{tt} = u_{xx} + u_{yy} + t\sin y \qquad -\infty < x,y < \infty \qquad t>0 \\ u\big|_{t=0} = x^2 \\ u_t\big|_{t=0} = \sin y \end{cases}$$

$$u = v + t\cdot\sin y.$$

$$\begin{cases} v_{tt} = v_{xx} + v_{yy} - t\cdot\sin y + t\sin y \\ v\big|_{t=0} + 0 = x^2 \\ v_t\big|_{t=0} + \sin y = \sin y \end{cases} \Rightarrow \begin{cases} v_{tt} = v_{xx} + v_{yy} \\ v\big|_{t=0} = x^2 \\ v_t\big|_{t=0} = 0 \end{cases}$$

$\exists\ v = x^2 + w(t)$

$$\begin{cases} w_{tt} = 1 \\ w(0) = 0 \\ w_t(0) = 0 \end{cases}$$

$$w = \frac{t^2}{2}$$

$$\Rightarrow \boxed{u = t\cdot\sin y + x^2 + \frac{t^2}{2}}$$

$$\begin{cases} u_{tt} = a^2 \Delta_3 u \\ u(r,0)=0 \\ u_t(r,0) = \begin{cases} v_0, & 0\le r\le 1 \\ 0, & r>1\end{cases}\end{cases}$$ – сферически сим. случай

Замена $v = r\cdot u$

$$\begin{cases} v_{tt} = a^2 v_{rr} \\ v(r,0)=0 \\ v_t(r,0) = \begin{cases} v_0 r, & 0\le r\le 1 \\ 0, & r>1\end{cases} \\ v(0,t)=0\end{cases}$$

– продливаем нечётным образом $\psi(r) = \begin{cases} v_0 r, & |r|\le 1\\ 0, & |r|>1\end{cases}$

$$v = \frac{1}{2a}\int\limits_{r-at}^{r+at}\psi(\rho)\,d\rho$$

II $v=0$

I $v = \frac{1}{2a}\int\limits_{r-at}^{r+at} v_0\rho\, d\rho = \frac{1}{2a}v_0\left(\frac{(r+at)^2}{2} - \frac{(r-at)^2}{2}\right) = \frac{v_0}{2a}\cdot\frac{2rat}{2}\cdot 2 = v_0 r t$

III $v = \frac{1}{2a}\int\limits_{r-at}^{1} v_0\rho\, d\rho = \frac{1}{2a}v_0\left(\frac12 - \frac{(r-at)^2}{2}\right)$

IV $v = \frac{1}{2a}\int\limits_{-1}^{1} v_0 \rho\, d\rho = 0$

$$u = \begin{cases} v_0 t, & r+at\le 1 \\ \frac{1}{2a}v_0\left(\frac12-\frac{(r-at)^2}{2}\right):r, & r+at>1,\ |r-at|\le 1 \\ 0, & r+at<1,\ |r-at|<1\end{cases}$$

$u(2,t) = ?$ При $r=2$ $r+at>1$

$2-at=1 \Rightarrow t=\frac1a$

$2-at=-1 \Rightarrow t=\frac3a$

$$u(2,t) = \begin{cases} 0, & t<\frac1a \\ \frac{1}{2a}v_0\left(\frac12-\frac{(2-at)^2}{2}\right):2, & \cancel{t\le\frac1a}\ \frac1a\le t\le\frac3a \\ 0, & t>3/a\end{cases} = \begin{cases} \frac{v_0}{8a}\left(-3+4at-(at)^2\right), & \frac1a\le t\le \\ 0, \text{ иначе}\end{cases}$$